

GCF

once the polynomial is in standard form, you need to find the GCF. The GCF is the largest factor among All terms, including numbers and variables.

GCF

$$\text{Ex: } 3x^2 - 15x^3 + 6x^4$$

$$\text{work: } 6x^4 - 15x^3 + 3x^2$$
$$3x^2(2x^2 - 5x + 1)$$

$$\text{Ex: } 5g^3h^3 - 10g^5h^2 + 20g^6h^4$$

$$\text{work: } 20g^6h^4 - 10g^5h^2 + 5g^3h^3$$
$$5g^3h^2(4g^3h^2 - 2g^2 + h)$$

COMMON FACTOR

once the polynomial is in standard form, the leading coefficient should be positive. If it is not positive, then find the common factor creating a positive leading coefficient.

COMMON FACTOR

$$\text{Ex: } 16x^2 - 15x^3$$

$$\text{work: } -15x^3 + 16x^2$$
$$-x^2(15x - 16)$$

$$\text{Ex: } 18a^3b - 9a^7b^3 - 21a^6b$$

$$\text{work: } -9a^7b^3 - 21a^6b + 18a^3b$$
$$-3a^3b(3a^4b^2 + 7a^3 - 6)$$

GCF or COMMON FACTOR

The Difference of Squares factors into the sum and difference pattern.

• Binomial?

• Subtraction?

• Perfect Squares?

If yes, then

$$a^2 - b^2 = (a+b)(a-b)$$

The sum or difference of 2 cubes factors into a

(Binomial) (Trinomial)

• Binomial?

• Perfect cubes?

If yes, then

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

or

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

The square of a sum and the square of a difference have products that are called Perfect Square Trinomials.

• Trinomial?

• First Term Positive?

• First Term Perfect Square?

• Last term positive?

• Last term perfect square?

• Does the Middle term = $a \cdot \sqrt{1st\ term} \cdot \sqrt{3rd\ term}$?

If yes then,

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

Trinomials

PERFECT SQUARE TRINOMIAL

$$Ex: 9x^2 - 30x + 25$$

Trinomial ✓

1st term positive ✓

3rd term positive ✓

1st term perfect sq ✓

3rd term perfect

square ✓

Middle term = $2 \cdot \sqrt{9x^2} \cdot \sqrt{25} \checkmark$

$$Ex: 9x^2 - 30x + 25 = (3x-5)^2$$

$$Ex: 4m^2 + 28mn + 49n^2$$

Trinomial ✓

1st term pos ✓

3rd term pos ✓

1st term perfect sq ✓

3rd term perfect

sq ✓

Middle = $2 \cdot \sqrt{4m^2} \cdot \sqrt{49n^2} \checkmark$

$$4m^2 + 28mn + 49n^2 = (2m+7n)^2$$

SPECIAL PRODUCTS

Leading coefficient must be 1

$$* \text{Remember } n_1 \cdot n_2 = c$$

$$n_1 + n_2 = b$$

$$(x+n_1)(x+n_2)$$

$$\text{EX: } x^2 + 4x - 12$$

$$n_1 \cdot n_2 = c \quad n_1 + n_2 = b$$

$$n_1 \cdot n_2 = -12$$

$$n_1 + n_2 = 4$$

$$-2 \cdot 6 = -12 \checkmark$$

$$-2 + 6 = 4 \checkmark$$

$$-2 \cdot 6 = -12 \checkmark$$

$$(x-2)(x+6)$$

$$\text{EX: } x^2 + 11xy + 10y^2$$

$$n_1 \cdot n_2 = c$$

$$n_1 + n_2 = b$$

$$n_1 \cdot n_2 = 10$$

$$n_1 + n_2 = 11$$

$$10 \cdot 1 = 10 \checkmark$$

$$10 + 1 = 11 \checkmark$$

$$(x+10y)(x+y)$$

$$x^2 + bx + c$$

Leading Coefficient should NOT be 1.

OPTION 1: GROUPING METHOD

STEPS:

- ① Identify a, b, c
- ② Find $n_1 \cdot n_2 = a \cdot c$ and $n_1 + n_2 = b$
- ③ Then rewrite the trinomial replacing b with n_1 and n_2 .
- ④ "Parenthesize"
- ⑤ Pull out a common factor from each ().
- ⑥ Regroup

OPTION 2:

$$ax^2 + bx + c \quad (\text{Part 1})$$

Ex: $5x^2 - 8x - 21$
 $a = 5 \quad b = -8 \quad c = -21$
 $n_1 + n_2 = a \cdot c \quad n_1 \cdot n_2 = b$
 $n_1 \cdot n_2 = -105 \quad n_1 + n_2 = -8$
 $7 + -15 = -8 \checkmark$
 $7 \cdot -15 = -105$

$$5x^2 - 8x - 21$$

$$5x^2 + 7x - 15x - 21$$

$$(5x^2 + 7x) + (-15x - 21)$$

$$x(5x + 7) - 3(5x + 7)$$

$$(x - 3)(5x + 7)$$

OPTION 2:

Ex

Ex: $18x^2 + 22xy - 28y^2$
 $2(9x^2 + 11xy - 14y^2)$
 $a = 9 \quad b = 11 \quad c = -14$
 $n_1 + n_2 = a \cdot c \quad n_1 \cdot n_2 = b$
 $n_1 \cdot n_2 = -126 \quad n_1 + n_2 = 11$
 $18 \cdot -7 = -126 \checkmark \quad 18 + -7 = 11 \checkmark$

$2(9x^2 + 17xy - 14y^2)$
 $9x^2 + 18xy - 7xy - 14y^2$
 $(9x^2 + 18xy) + (-7xy - 14y^2)$
 $9x(\cancel{x} + 2y) - 7y(\cancel{x} + 2y)$
 $2(9x - 7y)(x + 2y)$

$$ax^2 + bx + c$$

cont.

To factor 4 or more terms, grouping method:

* Practice: Factor the following by grouping

4 TERMS:

$$\text{D} 5x^3 - x^2 + 20x - 4$$
$$(5x^3 - x^2) + (20x - 4)$$
$$x^2(5x - 1) + 4(5x - 1)$$
$$(x^2 + 4)(5x - 1)$$

$$\textcircled{2} \quad xa - xb + 2a - 2b$$
$$(xa - xb) + (2a - 2b)$$
$$x(a - b) + 2(a - b)$$
$$(x + 2)(a - b)$$

$$\textcircled{3} \quad 2x^3 + 3x^2 - 8x - 12$$
$$(2x^3 + 3x^2) + (-8x - 12)$$
$$x^2(2x + 3) - 4(2x + 3)$$
$$(x^2 - 4)(2x + 3)$$
$$(x + 2)(x - 2)(2x + 3)$$

5 TERMS:

$$\textcircled{1} \quad x^4 + 2x^3 + x^2 + x + 1$$
$$(x^4 + 2x^3 + x^2) + (x + 1)$$
$$x^2(x^2 + 2x + 1) + (x + 1)$$
$$x^2(x + 1)^2 + (x + 1)$$
$$x^2(x + 1)(x + 1) + 1(x + 1)$$
$$(x + 1)[x^2(x + 1) + 1]$$

- Factor out a common
binomial factor of $(x + 1)$

Part 1

$$(x+1)(x^3+x^2+1)$$

$$\textcircled{2} \quad x^4 + x^3 + x^2 + 2x + 1$$

$$(x^4 + x^3) + (x^2 + 2x + 1)$$

$$x^3(x+1) + (x+1)^2$$

$$x^3(x+1) + (x+1)(x+1) \rightarrow \text{Factor out a common binomial factor}$$

$$(x+1)(x^3+x+1)$$

6 TERMS:

$$x^5 + x^4 + x^3 + x^2 + x + 1$$

$$(x^5 + x^4 + x^3) + (x^2 + x + 1)$$

$$x^3(\cancel{x^2+x+1}) + 1(\cancel{x^2+x+1})$$

$$(x^3+1)(x^2+x+1)$$

$$(x+1)(x^2-x+1)(x^2+x+1)$$

$$x^5 + x^4 + x^3 + x^2 + x + 1$$

$$\text{or } (x^5 + x^4) + (x^3 + x^2) + (x+1)$$

$$x^4(\cancel{x+1}) + x^2(\cancel{x+1}) + 1(\cancel{x+1})$$

$$(x^4 + x^2 + 1)(x+1)$$

$$*(x^2-x+1)(x^2+x+1) = (x^4 + x^2 + 1)$$

4 or MORE TERMS

Part 1